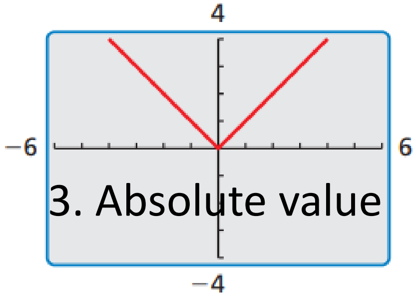
Today you will:

- Identify families of functions and the associated parent functions
- Describe transformations of parent functions
- Describe combinations of transformations
- Practice using English to describe math processes and equations

Core vocabulary:

- Parent function
- Transformation (of a graph of a function)
- Translation (of a graph of a function)
- Reflection (of a graph of a function)
- Vertical Stretch (of a graph of a function)
- Vertical Shrink (of a graph of a function)

a.



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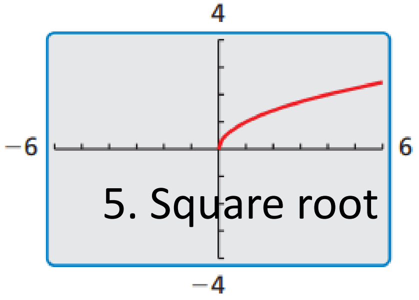

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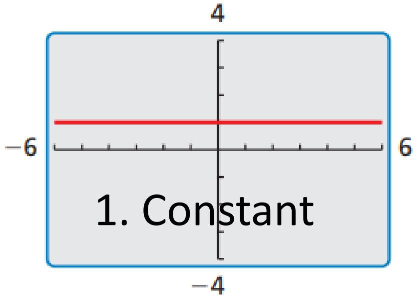

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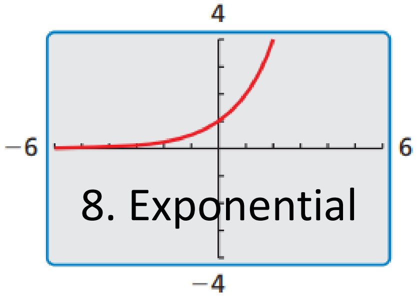

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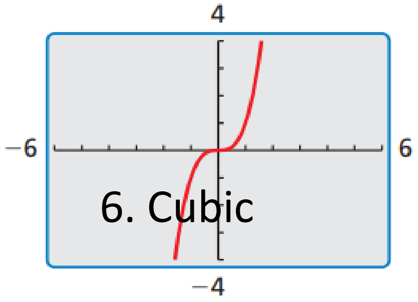

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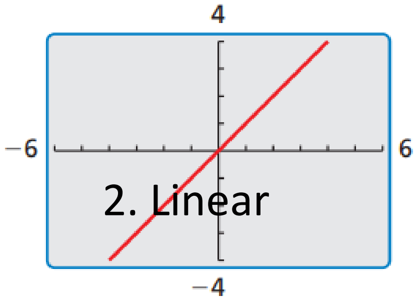
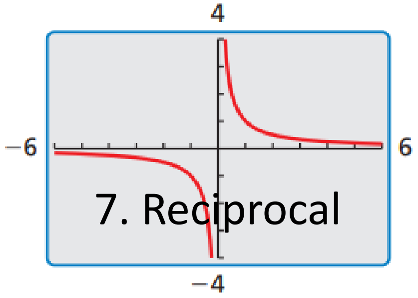
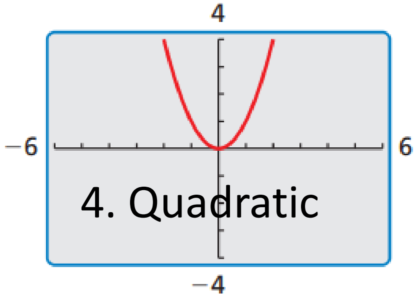


g.



h.



Match the graph on the left with the appropriate classification from the right

These graphs are graphs of basic “parent” functions

1. Constant
2. Linear
3. Absolute value
4. Quadratic
5. Square root
6. Cubic
7. Reciprocal
8. Exponential

What is a parent function?

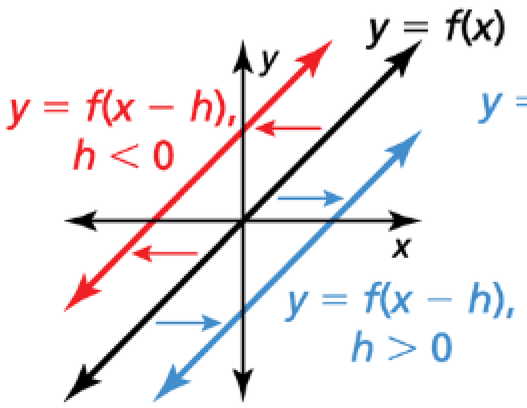
- The most basic function in a family of functions
  - For linear functions, the parent function is $f(x) = x$
  - For absolute value functions, the parent function is $f(x) = |x|$
  - For quadratic functions, the parent function is $f(x) = x^2$

What is a **transformation** (of a graph of a function)?

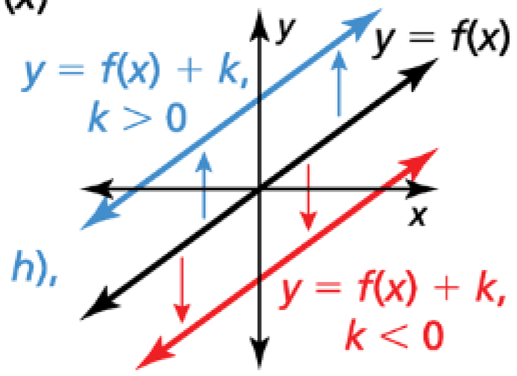
- A change in the size, shape, position or orientation of a graph.
- Example transformations:
  - Translation
  - Reflection
  - Vertical stretch
  - Vertical shrink

What is a **translation** (of a graph of a function)?

- A transformation that shifts a graph horizontally and/or vertically but does not change the size, shape, or orientation of the graph.



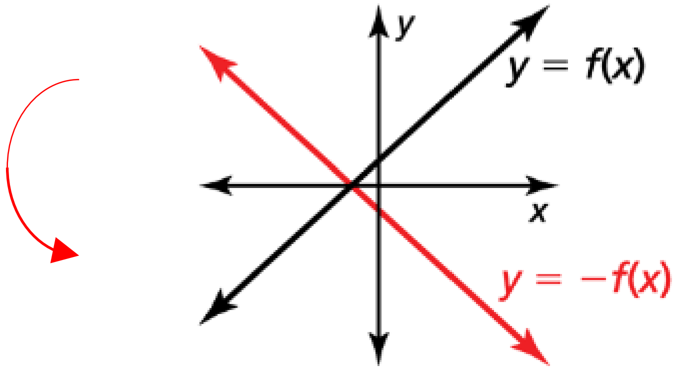
horizontal
translation



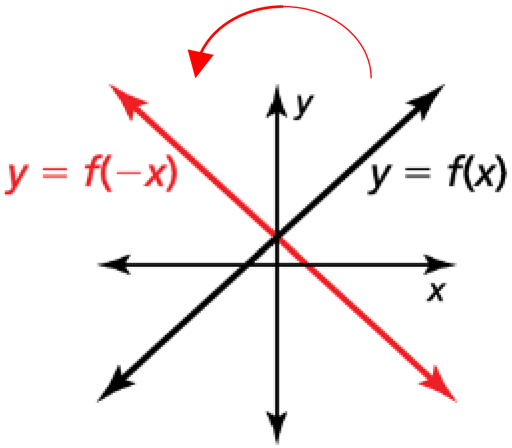
vertical
translation

What is a **reflection** (of a graph of a function)?

- A transformation that flips a graph over a line (called the *line of reflection)*.



Line of reflection is the x-axis

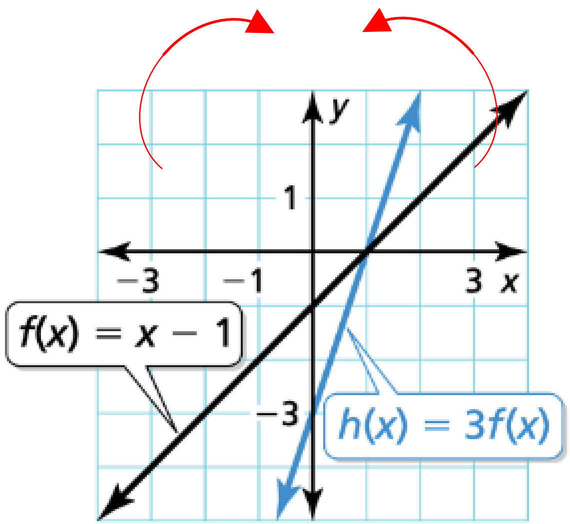


Line of reflection is the y-axis

What is a **vertical stretch** (of a graph of a function)?

- A transformation that causes the graph of a function to stretch away from the x-axis when all the y-coordinates are multiplied by a factor *a* where *a* > 1.

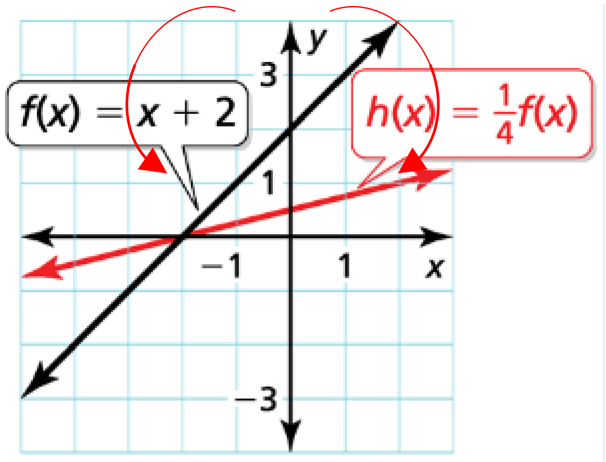
The graph of *h* is a vertical stretch of the graph of *f* by a factor of 3.



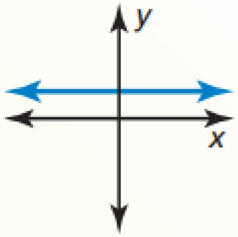
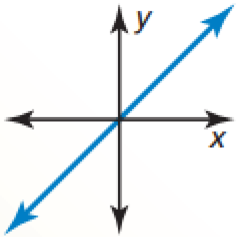
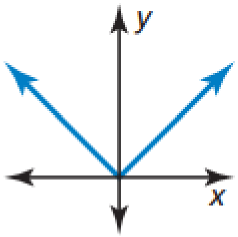
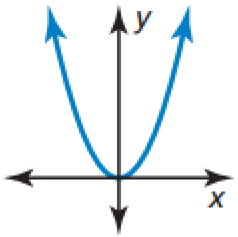
What is a **vertical shrink** (of a graph of a function)?

- A transformation that causes the graph of a function to stretch toward the x-axis when all the y-coordinates are multiplied by a factor *a* where 0 < *a* < 1.

The graph of h is a vertical shrink of the graph of f by a factor of $\frac{1}{4}$.



Function families and associated parent functions we will be working with today:

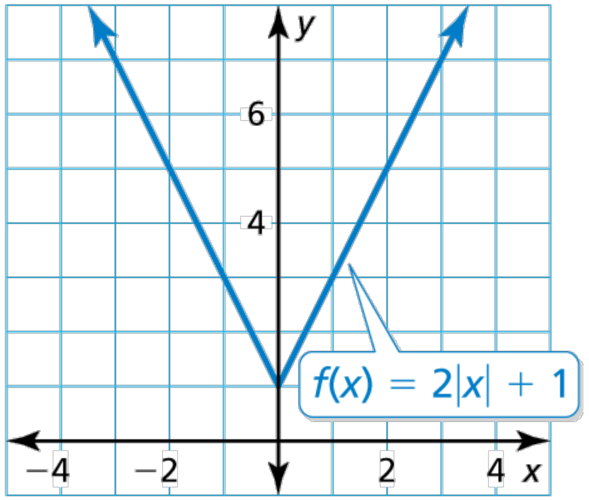
Family/name	Constant	Linear	Absolute Value	Quadratic
Parent function	$f(x) = 1$	$f(x) = x$	$f(x) = \|x\|$	$f(x) = x^2$
Graph				
Domain	All real numbers	All real numbers	All real numbers	All real numbers
Range	$y = 1$	All real numbers	$y \geq 0$	$y \geq 0$

Example 1

LOOKING FOR STRUCTURE

You can also use function rules to identify functions. The only variable term in f is an $|x|$ -term, so it is an absolute value function.

Identify the function family to which f belongs. Compare the graph of f to the graph of its parent function.



SOLUTION

The graph of f is V-shaped, so f is an absolute value function.

The graph is shifted up and is narrower than the graph of the parent absolute value function. The domain of each function is all real numbers, but the range of f is $y \geq 1$ and the range of the parent absolute value function is $y \geq 0$.

Example 2

REMEMBER

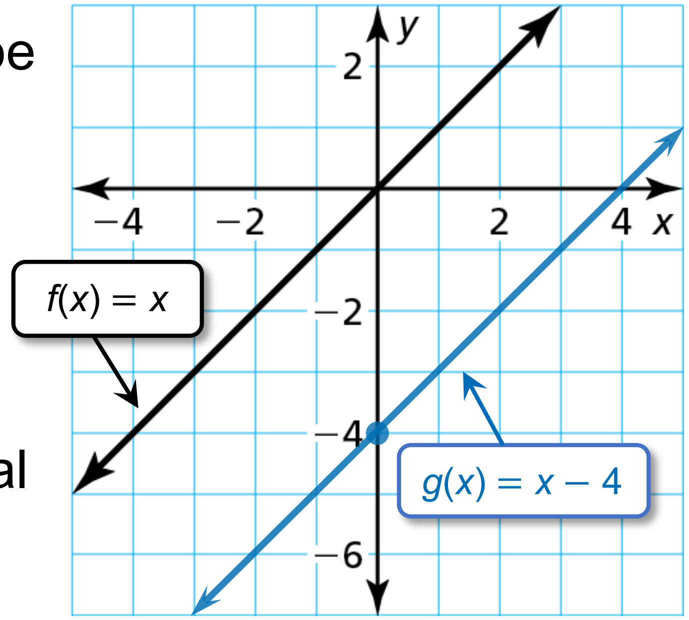
The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the y-intercept.

Graph $g(x) = x - 4$ and its parent function. Then describe the transformation.

SOLUTION

The function g is a linear function with a slope of 1 and a y-intercept of -4.

The graph of g is 4 units below the graph of the parent linear function f.



▶ So, the graph of $g(x) = x - 4$ is a vertical translation 4 units down of the graph of the parent linear function.

Example 3

Graph $p(x) = -x^2$ and its parent function. Then describe the transformation.

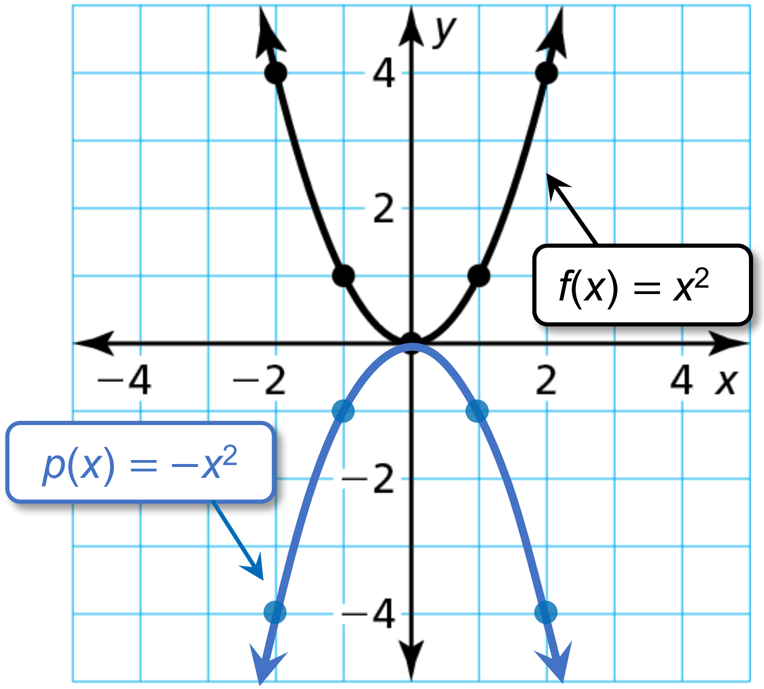
REMEMBER

The function $p(x) = -x^2$ is written in *function notation*, where $p(x)$ is another name for y.

SOLUTION

The function p is a quadratic function. Use a table of values to graph each function.

x	$y = x^2$	$y = -x^2$
−2	4	−4
−1	1	−1
0	0	0
1	1	−1
2	4	−4



The graph of p is the graph of the parent function flipped over the x-axis.

So, $p(x) = -x^2$ is a reflection in the x-axis of the parent quadratic function.

Example 4

Graph each function and its parent function. Then describe the transformation.

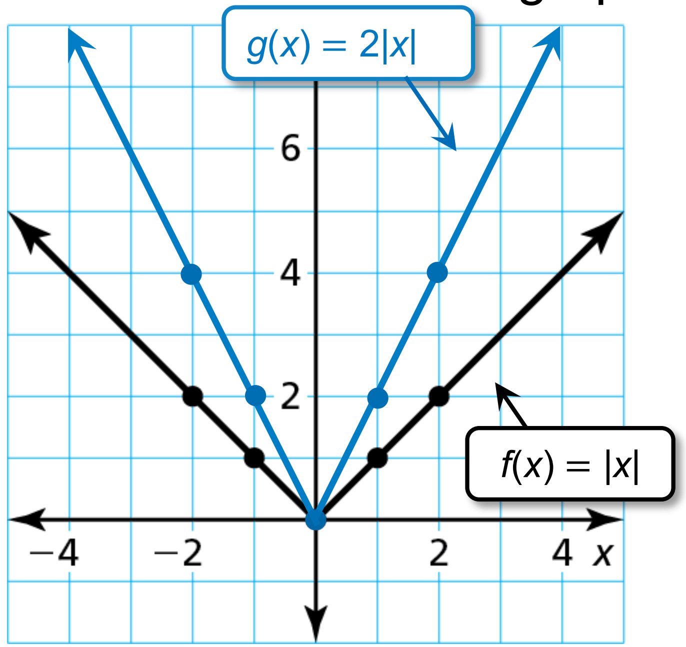
a. $g(x) = 2|x|$

b. $h(x) = \frac{1}{2}x^2$

SOLUTION

a. The function g is an absolute value function. Use a table of values to graph the functions.

x	$y = \lvert x \rvert$	$y = 2\lvert x \rvert$
−2	2	4
−1	1	2
0	0	0
1	1	2
2	2	4

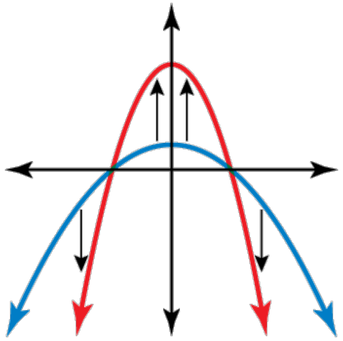


The y-coordinate of each point on g is two times the y-coordinate of the corresponding point on the parent function.

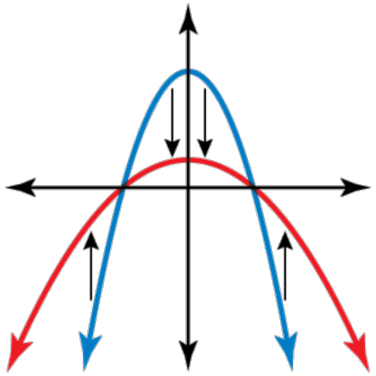
So, the graph of $g(x) = 2|x|$ is a vertical stretch of the graph of the parent absolute value function.

REASONING ABSTRACTLY

To visualize a vertical stretch, imagine *pulling* the points away from the *x*-axis.

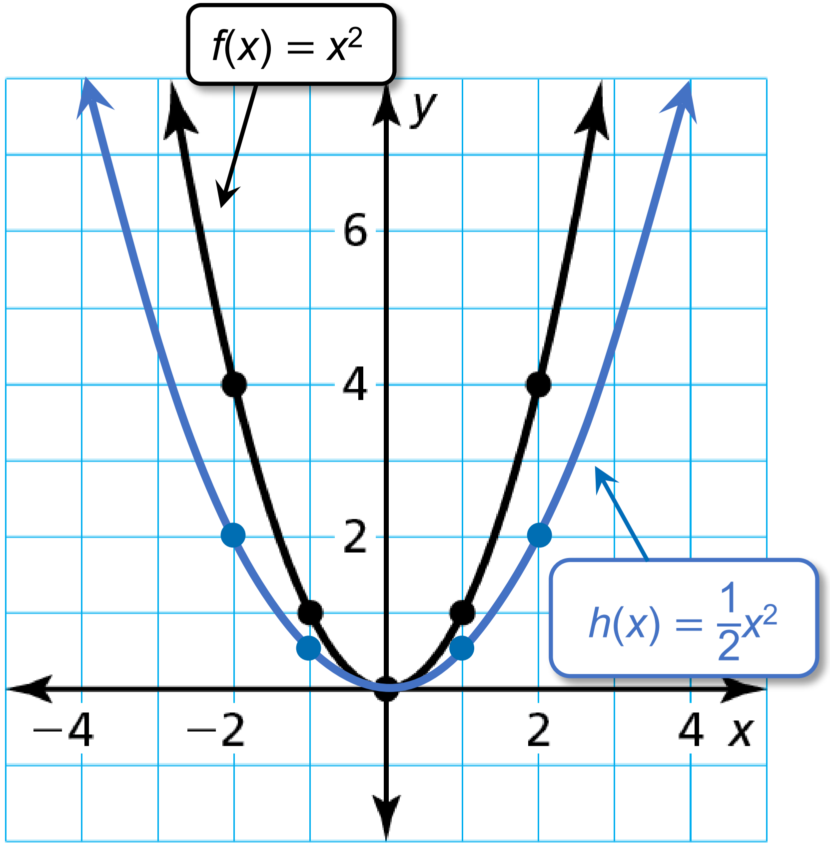


To visualize a vertical shrink, imagine *pushing* the points toward the *x*-axis.



b. The function *h* is a quadratic function. Use a table of values to graph the functions.

x	$y = x^2$	$y = \frac{1}{2}x^2$



The *y*-coordinate of each point on *h* is one-half of the *y*-coordinate of the corresponding point on the parent function.

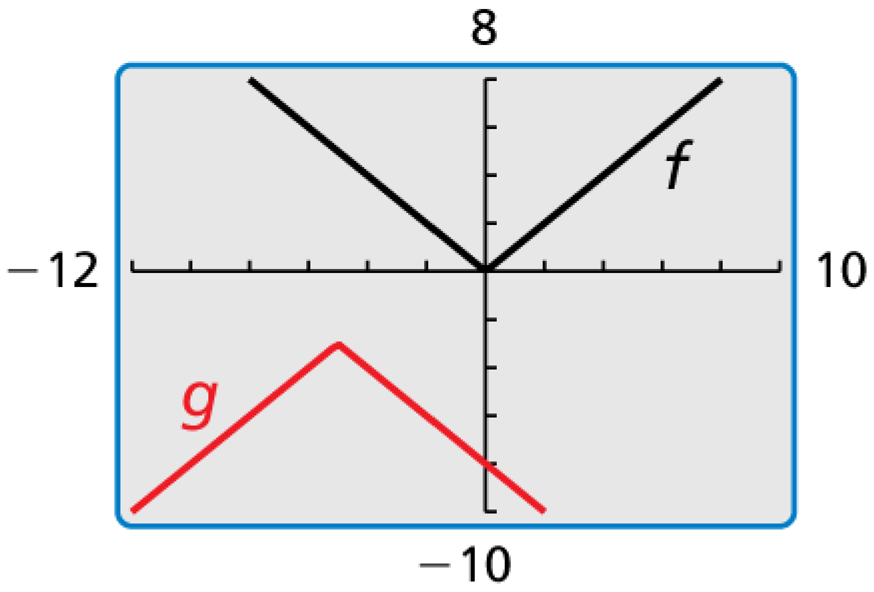
▶ So, the graph of $h(x) = \frac{1}{2}x^2$ is a vertical shrink of the graph of the parent quadratic function.

Example 5

Use a graphing calculator to graph $g(x) = -|x + 5| - 3$ and its parent function. Then describe the transformations.

SOLUTION

The function g is an absolute value function.



- The graph shows that $g(x) = -|x + 5| - 3$ is a reflection in the x-axis followed by a translation 5 units left and 3 units down of the graph of the parent absolute value function.

Example 6

Time (seconds), *x*	Height (feet), *y*
0	8
0.5	20
1	24
1.5	20
2	8

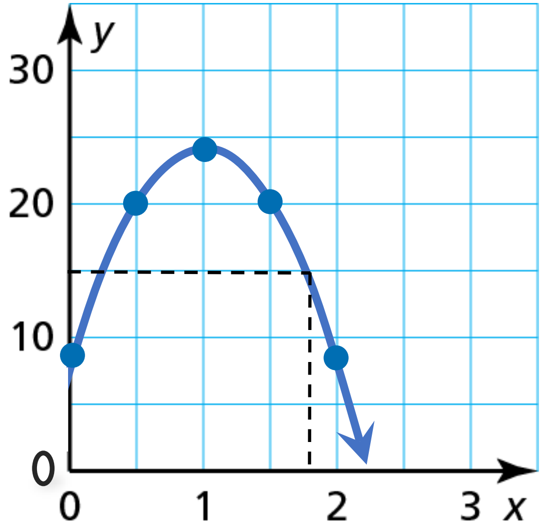
The table shows the height *y* of a dirt bike *x* seconds after jumping off a ramp. What type of function can you use to model the data? Estimate the height after 1.75 seconds.

SOLUTION

1. **Understand the Problem** You are asked to identify the type of function that can model the table of values and then to find the height at a specific time.

2. **Make a Plan** Create a scatter plot of the data. Then use the relationship shown in the scatter plot to estimate the height after 1.75 seconds.

3. **Solve the Problem** Create a scatter plot.



The data appear to lie on a curve that resembles a quadratic function. Sketch the curve.

So, you can model the data with a quadratic function. The graph shows that the height is about 15 feet after 1.75 seconds.

4. **Look Back** To check that your solution is reasonable, analyze the values in the table. Notice that the heights decrease after 1 second. Because 1.75 is between 1.5 and 2, the height must be between 20 feet and 8 feet.

$$8 < 15 < 20 \checkmark$$

Homework:

pg 8 #3-6, 9-18, 19-33 odd